



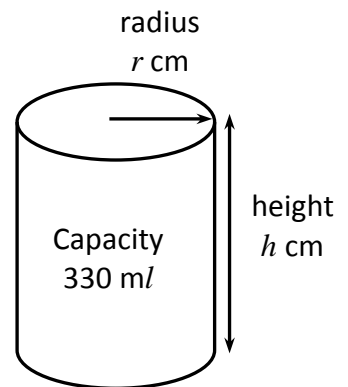
This activity is about using graphs to solve some maximum and minimum problems which occur in industry and in working life. The graphs can be drawn using a graphic calculator or a spreadsheet.

Information sheet

This example shows how to minimise the cost of making a can to hold a drink.

A typical can of drink holds 330 ml of liquid.

In order to keep the costs as low as possible, the manufacturer wants to use dimensions which make the surface area of the can as small as possible.



Think about ...

Which formulae do you think will be needed to solve this problem?

The can will hold 330 ml of drink – so the formula for volume is likely to be useful.

The manufacturer needs to find the radius and height of the cylindrical can that would have the least surface area – so the formula for surface area will also be needed.

The volume and surface area formulae for a cylinder are:

Volume: $V = \pi r^2 h$ **Surface area:** $S = 2\pi r^2 + 2\pi r h$

Since the volume = 330 ml = 330 cm³

$$330 = \pi r^2 h \text{ which rearranges to give } h = \frac{330}{\pi r^2}$$

Substituting for h in the formula for the surface area gives a formula for S in terms of just one variable, r :

$$S = 2\pi r^2 + 2\pi r \times \frac{330}{\pi r^2} \text{ which simplifies to } S = 2\pi r^2 + \frac{660}{r}$$

Excel or a graphic calculator can be used to draw a graph of S against r .

The Excel formula giving values of r at intervals of 0.1 cm and the corresponding values of S are shown below. Use this formula or enter the

formula $y = 2\pi x^2 + \frac{660}{x}$ into your graphic calculator to draw a graph of S against r for values of r from 0.1 cm to 10 cm.

	A	B
1	r	S
2	0.1	=2*PI()*A2^2+660/A2
3	=A2+0.1	=2*PI()*A3^2+660/A3
4	=A3+0.1	=2*PI()*A4^2+660/A4
5	=A4+0.1	=2*PI()*A5^2+660/A5
6	=A5+0.1	=2*PI()*A6^2+660/A6
7	=A6+0.1	=2*PI()*A7^2+660/A7
8	=A7+0.1	=2*PI()*A8^2+660/A8
9	=A8+0.1	=2*PI()*A9^2+660/A9
10	=A9+0.1	=2*PI()*A10^2+660/A10

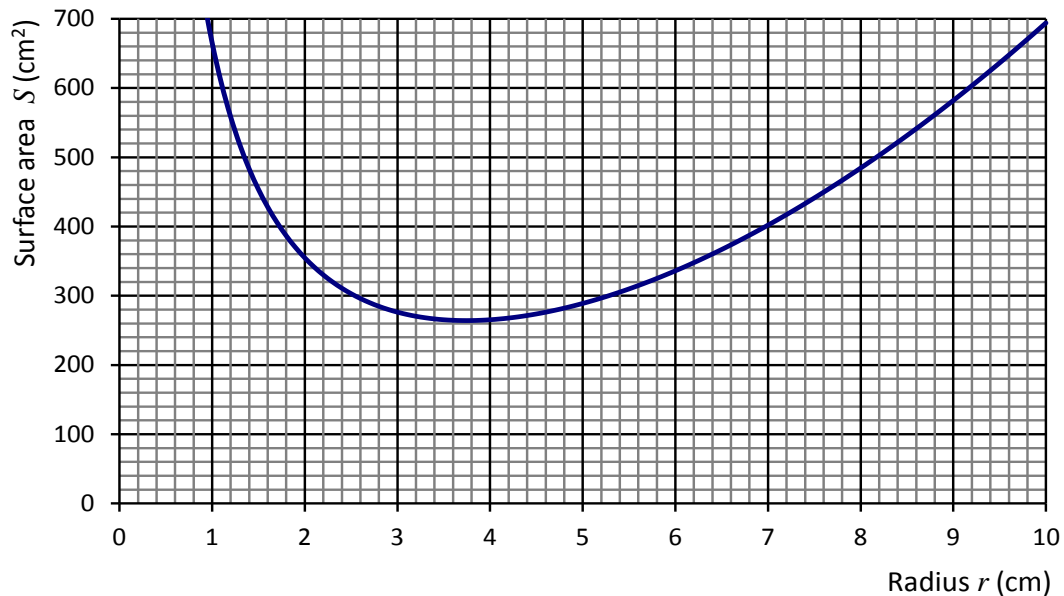
Note that you only need to enter 0.1 into A2 and the formulae shown into B2 and A3, then use 'fill down' to complete the rest of the table down to $r = 10$.

The graph you should get is shown below.

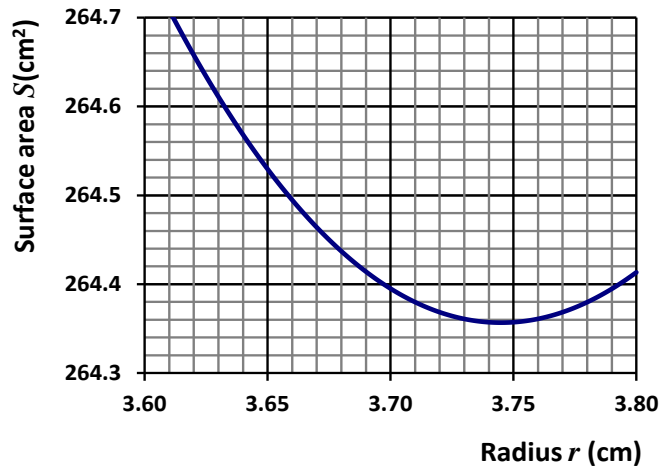
The graph shows that the minimum surface area is about 260 cm^2 . This will occur when the radius is approximately 3.7 cm. You can also see this from the table of values in Excel.

The same result can be obtained from a graphic calculator by using the Zoom and Trace functions near the minimum point on the graph.

Surface area of a can



Surface area of a can



More accurate values can be obtained by using smaller increments in the value of r in Excel, or by zooming in further on your graphic calculator.

Try this to obtain the values:

$$r = 3.745 \text{ and } S = 264.4$$

The can's height can be found from $h = \frac{330}{\pi r^2}$.

Substituting $r = 3.745$ gives $h = 7.490$

The can with the minimum surface area has radius 3.74 cm and height 7.49 cm (to 3 sf).

Now check that these dimensions do give a volume of 330 cm³ as required.

Try these

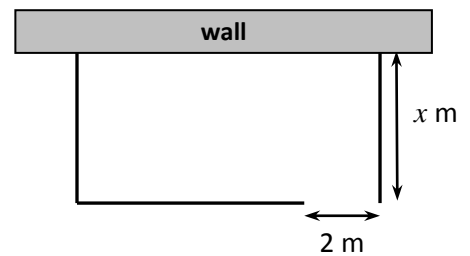
Use a graphic calculator or spreadsheet to solve these problems.

1a A soft drinks manufacturer wants to design a cylindrical can to hold half a litre (500 cm³) of drink.

Find the minimum area of material that can be used to make the can, and the corresponding dimensions of the can.

b Repeat part **a** for a can to hold 1 litre of drink.

2 A farmer has 100 metres of fencing to make a rectangular enclosure for sheep as shown. He will use an existing wall for one side of the enclosure, and leave an opening of 2 metres for a gate.



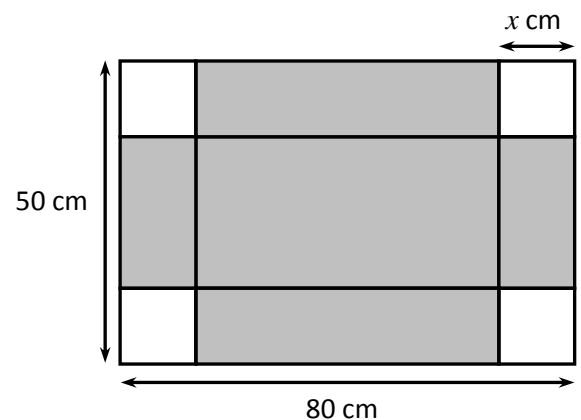
a Show that the area of the enclosure is given by
 $A = 102x - 2x^2$

b Find the maximum possible area, and the value of x that gives this area.

3 An open-topped box is to be made by removing squares from each corner of a rectangular piece of card, and then folding up the sides.

a Show that:
 if the original rectangle of card measured 80 cm by 50 cm, and the squares removed from the corners have sides x cm long, then the volume of the box is given by:

$$V = 4x^3 - 260x^2 + 4000x$$



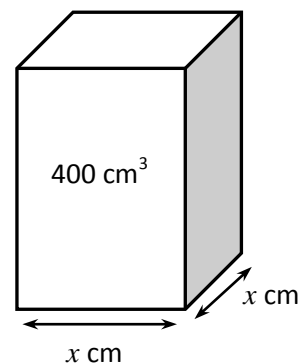
b Find the maximum possible volume, and the corresponding value of x .

4a A **closed** tank is to have a square base and capacity 400 cm^3

a i Show that the total surface area of the container is given by

$$S = 2x^2 + \frac{1600}{x}$$

a ii Find the minimum surface area and the value of x that will give this surface area.



b Find the minimum surface area of an **open-topped** tank with a square base and capacity 400 cm^3 and the dimensions of the tank with this surface area.

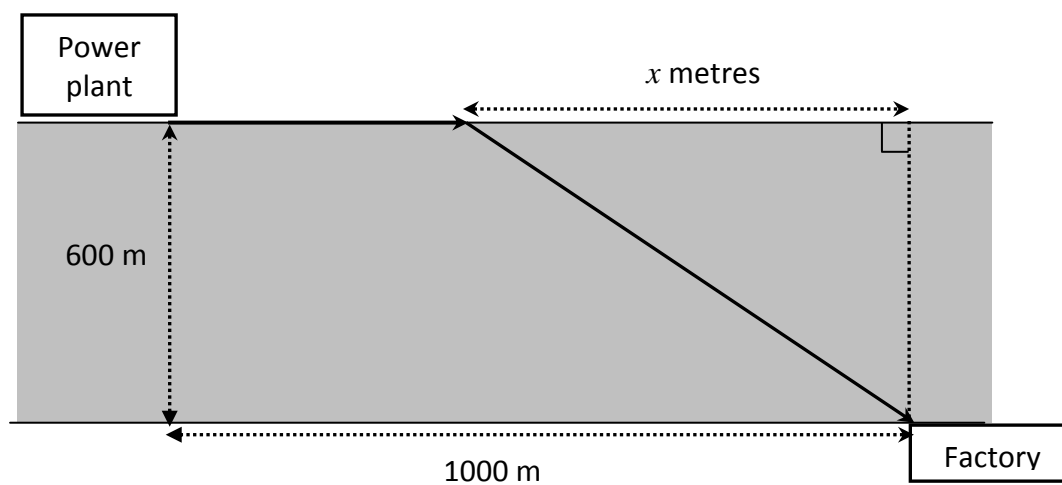
5 An underground power line is to run from a power plant at one side of a river to a factory at the other side, 1000 metres downstream.

The river is 600 m wide and has straight banks.

The sketch below shows the proposed route of the power line. It follows the river bank for some distance before crossing the river to the factory.

The cost of running the line under land is £40 per metre, and the cost under water is £50 per metre. It is required to find the route costing the least.

- Find the total cost of the line in terms of x .
- Find the route that gives the minimum cost.
- What is the minimum cost?



Reflect on your work

- Give a brief outline of the method used to find the minimum surface area for a can holding 330 ml of drink.
- What difference would it make to the surface area if a cuboid with square cross section was used to hold the 330 ml of drink?
- Do you think a cylinder is the best shape to use? Why?
- Can you find any connections between the types of equation that lead to a maximising problem and those which lead to a minimising problem?